Review for Math 334 – Midterm 2

Chapter 3

**Important skills:**

1. Be able to determine if a second order differential equation is linear or nonlinear, homogeneous, or nonhomogeneous. (If it can be put into the form given by Equation (3) in page 138, it is linear.)

2. Can you recognize a homogeneous equation with constant coefficients, and derive the characteristic equation? (Ex. 3, o. 149) This equation will be quadratic, so know the quadratic formula, the types of solutions one gets: real and distinct, repeated, and complex conjugate. These three cases will be crucial to the types of solutions one gets to constant coefficient homogeneous differential equations.

3. Be able to write down fundamental solution sets to homogeneous equations. This means find two solutions. (Ex. 3, p. 149)

4. What are the fundamental solution sets for each of the three cases of roots when solving constant coefficient equations? The summary is on p. 170. (Ex. 3, p. 149; Ex. 2, p. 169; Ex. 3, p. 162)

5. Solutions to second order nonhomogeneous equations have two components. There is the homogeneous solution, and particular or nonhomogeneous solution. (Thm. 3.5.2, p. 174) To find particular solutions you must know the method of undetermined coefficients, and variation of parameters. (Ex. 4, p. 178; Ex. 1, p. 185)

6. Mechanical vibrations give excellent examples for utilizing all the techniques in the Chapter. Know the difference between damped and undamped vibrations, forced and unforced situations.

7. For the unforced case, if there is no dampening, the motion is sinusoidal. Be able to determine the natural spring frequency. (Ex. 2, p. 196) If there is dampening, know the different kinds: underdamped, critically damped, and overdamped, depending on the roots to the characteristic equation. If underdamped, know the quasi period. (Ex. 3, p. 199) Know how to graph solutions in the three different cases of dampening.

**Relevant Applications:**

1. Mechanical Vibrations
2. Electric circuits

Chapter 4

**Definitions:**

1. Fundamental set of solutions (p. 221)
2. General solution (p. 221)
3. Homogeneous nth order linear equation (p. 220 equation (4))
4. Nonhomogeneous nth order linear equation (p. 219 equation (2))
5. Linearly dependent functions (p. 221)
6. Linearly independent functions (p. 221)
7. Characteristic equation for an nth order linear homogeneous constant coefficient equation (p. 226)
8. Wronskian for n functions (p. 221)

**Theorems:**
1. Theorem 4.1.2: General solutions to higher order linear ODE’s and the fundamental set of solutions (p. 221)
2. Theorem 4.1.3: Related linear independence to fundamental sets of solutions. (p. 223)

**Important skills:**
1. The methods for solving higher order linear differential equations are extremely similar to those of the last chapter. The general solution to an nth order homogeneous linear differential equation is obtained by linearly combining n linearly independent solutions. (Eq. (5), p. 220)
2. The generalization of the Wronskian is given on page 221. It is used as in the last Chapter to show the linear independence of functions, and in particular, homogeneous solutions.
3. For the situation where there are constant coefficients, you should be able to derive the characteristic polynomial, and the characteristic equation, in this case of nth order. Depending on the types of roots you get to this equation you will have solution sets containing functions similar to those in the second order case. (Ex. 2-4, p. 229-231)
4. The general solution of the nonhomogeneous problem easily extends to the nth order case. (Eq. (9), p. 227)
5. Both variation of parameters, and the method of undetermined coefficients generalize to determine particular solutions in the higher dimensional situation. (Ex. 3, p. 236; Ex. 1, p. 241)

**Relevant Applications**
1. Double and multiple spring mass systems

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**Chapter 5**

**Definitions:**
1. Ordinary point for a second order linear equation (p. 250)

**Theorems:**
1. Theorem 5.3.1: Existence of series solutions to linear ODE’s near ordinary points and their convergence properties. (p. 262)

**Important skills:**

1. Review power series, how to shift the index of summation, (Ex. 3, p. 247) and tests for convergence.
2. Know how to find the interval of convergence for a power series. (Ex. 2, p. 245)
3. Be able to determine all ordinary and singular points for a differential equation. (p. 250-1)
4. For ordinary points, Eq. (3) on page 251 gives the form of the solution. Be able to derive the recursion relation, as in Example 1. If the recursion relation can be solved, one obtains the two solutions of the homogeneous problem. (Ex. 1, p. 251)
5. The method described in the second paragraph on page 244 can be used to find the first several terms in each of the homogeneous solutions.
6. Be able to determine lower bounds on the radius of convergence of the series solutions. (Ex. 4, p. 264)