Homework Assignment 10

January 26, 2011

1. Prove that the Möbius bundle is a smooth vector bundle.

2. For $M$ a manifold and $\{(U_\alpha, \varphi_\alpha)\}$ and $\{(U'_{\beta}, \varphi'_{\beta})\}$ two orientations we say they are equivalently oriented if $\varphi'_{\beta} \circ \varphi^{-1}_{\alpha}$ has positive Jacobian determinant. Show this is an equivalence relation.

3. Let $f : M \rightarrow N$ be a diffeomorphism of connected oriented manifolds. Show that if $df_x : T_xM \rightarrow T_{f(x)}N$ preserves orientation at one point $x$, then $f$ preserves orientation globally.