Homework Assignment 7

January 18, 2011

1. Let $M$ be a $k$-dimensional smooth manifold and $S(M)$ be point $(x, v) \in TM$ such that $|v| = 1$. Prove that $S(M)$ is a $2k - 1$-dimensional subbundle of $TM$ called the sphere bundle of $M$.

2. Let $G : \mathbb{R}^2 \to \mathbb{R}^4$ be given by

$$G(x, y) = ((r \cos y + a) \cos x, (r \cos y + a) \sin x, r \sin y \cos \frac{x}{2}, r \sin y \sin \frac{x}{2}).$$

Show this gives an embedding of the Klein bottle.

3. Show that $f : S^1 \to \mathbb{R}^2$ given by $f(\cos t, \sin t) = (\sin(2t) \cos t, \sin(2t) \sin t)$ is an immersion. Explain why $f(S^1)$ is not a submanifold of $\mathbb{R}^2$. 