Homework Assignment 9

January 25, 2011

1. Prove that $[V, [W, X]] + [W, [X, V]] + [X, [V, W]] = 0$ and for all $f, g \in C^\infty(M)$ we have $[fV, gW] = fg[V, W] + (fVg)W - (gWf)V$.

2. Show that $\text{Lie}(\mathbb{T}^n) \simeq \mathbb{R}^n$.

3. Find $[V, W]$ in $\mathbb{R}^3$ for the following:
   (a) $V = y \frac{\partial}{\partial z} - 2xy^2 \frac{\partial}{\partial y}$ and $W = \frac{\partial}{\partial y}$.
   (b) $V = x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x}$ and $W = y \frac{\partial}{\partial z} - z \frac{\partial}{\partial y}$.
   (c) $V = x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x}$ and $W = x \frac{\partial}{\partial y} + y \frac{\partial}{\partial x}$.

4. Show that $\mathbb{R}^3$ with the cross product is a Lie algebra.