3.4.5 (4 pts) Let \( A, B \subset \mathbb{R} \) such that there exist \( U \) and \( V \) open subsets of \( \mathbb{R} \) where \( A \subset U \), \( B \subset V \), and \( U \cap V = \emptyset \). We know that \( \overline{A} \subset U \), \( \overline{B} \subset V \), \( U \cap V = \emptyset \), and \( U \cap V = \emptyset \). So \( \overline{A} \cap B = \emptyset = A \cap \overline{B} \). Hence, \( A \) and \( B \) are separated.

3.4.7(a) (2 pts) Let \( A = \mathbb{Q} \). Then \( \mathbb{Q} \) is disconnected, but \( \overline{A} = \mathbb{R} \) is connected.

3.4.7(b) (5 pts) Let \( A \) be connected. So for all nonempty sets \( C \) and \( D \) in \( \mathbb{R} \) such that \( A \subset C \cup D \) we know that \( \overline{C} \cap D \) or \( C \cap \overline{D} \) is nonempty. For \( \overline{A} \) if \( E \) and \( F \) are subset of \( \mathbb{R} \) such that \( A \subset E \cup F \) we know that \( A \subset E \cup F \). So either \( \overline{E} \cap F \) or \( E \cap \overline{F} \) is nonempty. Hence, \( \overline{A} \) is connected.

If \( A \) is perfect, then \( A \) is closed so \( A = \overline{A} \) and \( A \) is connected. So \( \overline{A} \) is perfect. \( \square \)

3.4.8(a) (3 pts) Let \( x, y \in \mathbb{Q} \). Assume that \( x < y \). Fix \( r \in \mathbb{I} \) such that \( x < r < y \). Let \( A = \{ q \in \mathbb{Q} : q < r \} \) and \( B = \{ q \in \mathbb{Q} : q > r \} \). Then \( x \in A \), \( y \in B \), and \( \overline{A} \cap B = \emptyset = A \cap \overline{B} \).

3.4.9(a) (2 pts) Given \( x, y \in C \) with \( x < y \) fix \( \epsilon = y - x \). Then there exists some \( N \in \mathbb{N} \) such that \( 1/3^N < \epsilon \). then \( x \) and \( y \) are in separate intervals of length \( 1/3^n \) for all \( n \geq N \). Hence, \( x \) and \( y \) are in different components of \( C_n \) for all \( n \geq N \).

3.4.9(b) (2 pts) Now let \( z \in [x, y] \) such that \( z \notin C_N \). Then \( z \notin C \). If \( a, b \in C \), then for \( \epsilon = b - a \) we see that there exists some \( z \in (a, b) \) such that \( z \notin C \). So \( (a, b) \) is not a subset of \( C \).

3.4.9(c) (2 pts) Let \( x, y \in C \) and \( z \in (x, y) \) such that \( z \notin C \). Define \( A = C \cap (-\infty, z] \) and \( B = C \cap [z, \infty) \). Then \( A \) and \( B \) are nonempty, \( A \cup B = C \), and \( A \) and \( B \) are separated since both are closed.