4.2.1 (8 pts)

a) Fix $\epsilon > 0$. Let $\delta = \epsilon / 2$. For $0 < |x - 2| < \delta$ we have

$$|2x + 4 - 8| = |2x - 4| = 2|x - 2| < 2(\epsilon / 2) = \epsilon.$$ 

Hence, $\lim_{x \to 2} 2x + 4 = 8$.

b) Fix $\epsilon > 0$. Let $\delta = \sqrt[3]{\epsilon}$. For $0 < |x - 0| < \delta$ we have

$$|x^3 - 9| = |x^3| < (\sqrt[3]{\epsilon})^3 = \epsilon.$$ 

So $\lim_{x \to 0} x^3 = 0$.

c) Fix $\epsilon > 0$. Let $\delta = \min\{1, \epsilon / 19\}$. For $0 < |x - 2| < \delta$ we have

$$|x^3 - 8| = |x - 2||x^2 + 2x + 4| < (\epsilon / 19)(19) = \epsilon.$$ 

Therefore, $\lim_{x \to 2} x^3 = 8$.

d) Fix $\epsilon > 0$. Let $\delta = \frac{1}{10}$. For $0 < |x - \pi| < \delta$ we know that $\lfloor x \rfloor = 3$. Since $2 < \pi - \frac{1}{10} < x < \pi + \frac{1}{10} < 3.5$ we have $|\lfloor x \rfloor - 3| = 0 < \epsilon$.

4.2.3(a) (3 pts) For $x > 0$ we know that $\frac{|x|}{x} = \frac{x}{x} = 1$. So $\{x_n\} = \{\frac{1}{n}\}$ converges to 0 and $\lim_{n \to \infty} f(x_n) = 1$. For $x < 0$ we have $\frac{|x|}{x} = -\frac{x}{x} = -1$. So $\{y_n\} = \{-\frac{1}{n}\}$ converges to 0 and $\lim_{n \to \infty} f(y_n) = -1$. By Corollary 4.5 we know that $\frac{|x|}{x}$ does not have a limit at $x = 0$.

4.2.7 (4 pts)

a) Let $f(x)$ be defined in a neighborhood of $c \in \mathbb{R}$. Then $\lim_{x \to c} f(x) = \infty$ if for all $M \in \mathbb{R}^+$ there exists some $\delta > 0$ such that $f(x) > M$ for all $0 < |x - c| < \delta$.

b) $\lim_{x \to \infty} f(x) = L$ if for all $\epsilon > 0$ there exists some $M \in \mathbb{R}$ such that $|f(x) - L| < \epsilon$ for all $x > M$.

4.2.9 (5 pts) Fix $\epsilon > 0$. Then there exists some $\delta_1 > 0$ such that $0 < |x - c| < \delta_1$ implies that $|f(x) - L| < \epsilon$. Similarly, there exists some $\delta_2 > 0$ such that $0 < |x - c| < \delta_2$ implies that $|h(x) - L| < \epsilon$. Fix $\delta = \min\{\delta_1, \delta_2\}$. So if $0 < |x - c| < \delta$, then

$$L - \epsilon < f(x) \leq g(x) \leq h(x) < L + \epsilon$$

and $|g(x) - L| < \epsilon$. Hence, $\lim_{x \to c} g(x) = L$. □