4.6.3 (5 pts) Let \( \lim_{x \to c} f(x) = L \). Fix \( \epsilon > 0 \). Then there exists some \( \delta > 0 \) such that if \( 0 < |x - c| < \delta \), then \( |f(x) - L| < \epsilon \). So if \( 0 < x - c < \delta \) we have \( |f(x) - L| < \epsilon \) and if \( 0 < c - x < \delta \) we have \( |f(x) - L| < \epsilon \). Therefore, \( \lim_{x \to c^-} f(x) = L \) and \( \lim_{x \to c^+} f(x) = L \).

If \( \lim_{x \to c^-} f(x) = L \) and \( \lim_{x \to c^+} f(x) = L \), then there exist \( \delta_1, \delta_2 > 0 \) such that \( 0 < x - c < \delta_1 \) implies that \( |f(x) - L| < \epsilon \) and \( 0 < c - x < \delta_2 \) implies \( |f(x) - L| < \epsilon \).

Let \( \delta = \min\{\delta_1, \delta_2\} \). So if \( 0 < |x - c| < \delta \), then \( 0 < x - c < \delta \leq \delta_1 \) or \( 0 < c - x < \delta \leq \delta_2 \). Hence, \( |f(x) - L| < \epsilon \) and \( \lim_{x \to c} f(x) = L \). \( \Box \)

4.6.6 (5 pts) (i) Let \( F_1 = \mathbb{R} \). Since \( F_1 \) is closed we know that \( \mathbb{R} \) is the countable union of closed sets. (ii) Let \( F_i = \mathbb{R} - (\frac{-1}{i}, \frac{1}{i}) \). So each \( F_i \) is closed and \( \bigcup F_i = \mathbb{R} - \{0\} \). (iii) Enumerate \( \mathbb{Q} \). Then let \( F_i = \{q_i\} \). Each \( F_i \) is closed and \( \bigcup F_i = \mathbb{Q} \). (iv) Same as for \( \mathbb{Q} \) since \( \mathbb{Z} \) is countable. (v) Let \( F_i = [\frac{1}{i}, 1] \). Then \( \bigcup F_i = (0, 1] \).