6.6.2 (4 pts) We know that
\[
\frac{d}{dx} (\ln(1 + x)) = \frac{1}{1 + x} = \sum_{n=0}^{\infty} (-1)^n x^n.
\]

To obtain a power series for \(\ln(1 + x)\) we then integrate term by term and notice that \(\ln(1) = 0\) so \(a_0 = 0\). Then
\[
\ln(1 + x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}.
\]

6.6.6 (a) Since \(f^{(n)}(x) = e^x\) for all \(n \in \mathbb{N}\) we know that
\[
e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}.
\]

(b) Also
\[
\frac{d}{dx} e^x = \sum_{n=1}^{\infty} \frac{x^{n-1}}{(n-1)!} = \sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x.
\]

6.6.10 (5 pts) We have \(g'(x) = \frac{2}{x^3} e^{-1/x^2}\) for \(x \neq 0\), \(g''(x) = \left(\frac{-6}{x^4} + \frac{2}{x^3}\right) e^{-1/x^2}\) for \(x \neq 0\), and \(g'''(x) = \left(\frac{24}{x^5} + \frac{6}{x^4} + \frac{2}{x^3}\right) e^{-1/x^2}\) for \(x \neq 0\). For the general term we have
\[
g^{(n)}(x) = \left(\sum_{k=1}^{n} \frac{(-1)^{k+1}(k+1)!}{x^{k+2}}\right) e^{-1/x^2}
\]
for \(x \neq 0\).

6.6.11 (5 pts) We have
\[
g''(0) = \lim_{x \to 0} \frac{2}{x^3} e^{-1/x^2} - \frac{2}{x^3} = \lim_{x \to 0} \frac{2}{x^2} = \frac{2}{x^2}.
\]

Using l’Hôpital’s rule we get \(g''(0) = 0\). Now assume that \(g^{(k)}(0) = 0\). Then
\[
g^{(k+1)}(0) = \lim_{x \to 0} \frac{\left(\sum_{k=1}^{n} \frac{(-1)^{k+1}(k+1)!}{x^{k+2}}\right) e^{-1/x^2} - 0}{x - 0} = \lim_{x \to 0} \frac{\sum_{k=1}^{n} \frac{(-1)^{k+1}(k+1)!}{x^{k+2}}}{e^{1/x^2}}.
\]

Using l’Hôpital’s rule we get \(g^{(k+1)}(0) = 0\).