7.2.2 (a) \( L(f, P) = 3/2 + 2 + 5 = 8.5 \), \( U(f, P) = 2 + 5/2 + 7 = 11.5 \), and \( U(f, P) - L(f, P) = 3 \)

(b) \( L(f, P) = 3/2 + 2 + 5/2 + 3 = 9 \), \( U(f, P) = 2 + 5/2 + 3 + 7/2 = 11 \), and \( U(f, P) - L(f, P) = 2 \).

(c) Add any point to the partition \( \{1, 3/2, 2, 5/2, 3\} \) and the new partition \( P' \) will have \( U(f, P') - L(f, P') < 2 \).

7.2.4 (a) Suppose that \( \lim_{n \to \infty} [U(f, P_n) - L(f, P_n)] = 0 \). Since \( f \) is bounded on a compact interval we know that \( U(f) \) and \( L(f) \) exist. Then \( U(f) = L(f) \) and the function is integrable.

Now suppose that \( f \) is bounded on a compact interval and integrable. Then for \( 1/2^n \) we know that there exists some partition \( P_n \) such that \( U(f, P_n) - L(f, P_n) < 1/2^n \). So there exists a sequence of partitions where \( \lim_{n \to \infty} [U(f, P_n) - L(f, P_n)] = 0 \).

7.2.4 (b) We have

\[
L(f, P_n) = \frac{1}{n} \sum_{i=0}^{n-1} i \cdot \frac{n}{n} = \frac{1}{n} \frac{(n-1)n}{2} = \frac{n^2-n}{2n^2}
\]

and

\[
U(f, P_n) = \frac{1}{n} \sum_{i=1}^{n} i \cdot \frac{n}{n} = \frac{n^2+n}{2n^2}.
\]

So \( U(f, P_n) - L(f, P_n) = 1/n \).

7.2.4 (c) We know that

\[
\lim_{n \to \infty} U(f, P_n) - L(f, P_n) = \lim_{n \to \infty} \frac{1}{n} = 0.
\]

7.2.5 Assume that \( f_n \) is integrable on \([a,b]\) and \( f_n \to f \) uniformly. Then there exists some \( N \in \mathbb{N} \) such that \(|f_N(x) - f(x)| < \epsilon/(3|b-a|)\) for all \( x \in [a,b] \). Let \( P_N \) be a partition of \([a,b]\) such that \( U(f_N, P_N) - L(f_N, P_N) < \epsilon/3 \). Then

\[
U(f, P_N) \leq U(f_N, P_N) + \frac{\epsilon}{3|b-a|} |b-a| = U(f_N, P_N) + \frac{\epsilon}{3}
\]

and

\[
L(f, P_N) \geq L(f_N, P_N) - \frac{\epsilon}{3|b-a|} |b-a| = L(f_N, P_N) - \frac{\epsilon}{3}.
\]

So

\[
U(f, P_N) - L(f, P_N) = U(f, P_N) - U(f_N, P_N) + U(f_N, P_N) - L(f_N, P_N) + L(f_N, P_N) - L(f, P_N)
\]

\[
< \frac{\epsilon}{3} + \frac{\epsilon}{3} + \frac{\epsilon}{3} = \epsilon.
\]