1. Let $T : X \to Y$ be continuous. Show $T$ induces a natural map $M(X) \to M(Y)$ and that this map is continuous in the weak$^*$ topology.

2. Let $\alpha \in (0, 1)$ be irrational and $T : \mathbb{T}^d \to \mathbb{T}^d$ be defined by

$$T(x_1, \ldots, x_d) = (x_1 + \alpha, x_2 + a_{21}x_1, \ldots, x_k + a_{k1}x_1 + \cdots a_{kk-1}x_{k-1})$$

where the coefficients $a_{ij}$ are integers and $a_{ii} \neq 0$. Use a Fourier transformation to show that $T$ is ergodic with respect to Lebesgue. (Start with low dimension if this is easiest).

3. Let $(X, \mathcal{A}, \mu)$ and $(Y, \mathcal{B}, \nu)$ be measure spaces and $T : X \to Y$ be measure preserving. Prove that $U_T$ is surjective if and only if $\tilde{T} : \tilde{\mathcal{B}} \to \tilde{\mathcal{A}}$ is an isomorphism.

4. For a probability space $(X, \mathcal{A}, \mu)$ define a metric on $\tilde{\mathcal{A}}$ by $d(A, B) = \mu(A \triangle B)$.

   (a) Prove that $d(\cdot, \cdot)$ is indeed a metric and that with this metric the set $\tilde{\mathcal{A}}$ is complete.

   (b) Prove that $(X, \mathcal{A}, \mu)$ is separable if and only if $\tilde{\mathcal{A}}$ is separable.