Homework Assignment 9

March 27, 2013

1. Prove that any topologically transitive translation of the 2-torus has a one-sided generator. (Hint: It is sufficient to start with just 2-elements in the partition.)

2. Prove that the measure preserving transformations $\sigma_2 : \Sigma_2 \to \Sigma_2$ with product measure $(1/2, 1/2)$ and $\sigma_3 : \Sigma_3 \to \Sigma_3$ with product measure $(1/3, 1/3, 1/3)$ are not metrically isomorphic.

3. The Gauss transformation of $[0, 1)$ to itself is $G(x) = (1/x) - \lfloor 1/x \rfloor$ and an invariant probability measure is given by

$$\mu(E) = \frac{1}{\log 2} \int_E \frac{dx}{1 + x}.$$ 

Let $P$ be a partition given by $[1/(m+1), 1/m)$. Show that if $P^n = \bigvee_{j=0}^{n-1} G^{-j}(P)$ then there exists some $c_1, c_2 > 0$ such that $c_1 m(P_n) \leq \mu(P_n) \leq c_2 m(P_n)$ for all $n \geq 1$ and $P_n \in P^n$ where $m$ is Lebesgue measure. Furthermore, show that

$$\text{diam}(P_n) \leq \sup_{x \in P_n} \frac{1}{[G^n]'(x)} \leq 2^{-[n/2]}.$$ 

Using these facts prove that $h_\mu(G) = \int \log |G'| d\mu$. 

1