Math 634 Midterm - Review

October 29, 2014

The midterm covers section 1.1-4.6 in the course notes.

Chapter 1

Section 1.1 is mostly introductory you should know the definition of flow and that equivalence between the differential and integral equations.

Nothing will directly be asked from Section 1.2, but you need to know the facts including the statement of Ascoli’s Theorem and the Contraction Mapping Theorem.

Section 1.3 mostly introduces notation and general ideas of first order systems.

Section 1.4 proves the Cauchy-Peano existence Theorem (Theorem 1.10 in the notes). You should know the statement and proof of this theorem.

Section 1.5 proves the Picard-Lindelöf Theorem on uniqueness (Theorem 1.15 in the notes) you should know the statement and proof of this theorem. Also in this section we prove the Gronwall inequality. You should know the statement of this theorem (Theorem 1.14 in the notes).

The main result in Section 1.6 is Theorem 1.21 you should know this statement and methods from this section.

Section 1.7 proves that the solutions depend continuously on the initial conditions. The proof just uses the fact that \( f \) is locally Lipschitz.

There is nothing from Section 1.8 on the exam.

Chapter 2

In Section 2.1 we introduce some basic facts about matrices that you should know, but won’t be asked specifically. The main theorem states that if we have a unique solutions exist for all time.
Section 2.2 introduces fundamental matrices for linear systems. You should know the definition of a fundamental matrix as well as the proof that these exist. You also should know the statement and proof of Abel’s formula (Theorem 2.11 in the notes).

Section 2.3 introduces higher order linear systems. Much of the theory is similar to the previous sections. The exception is the introduction of the Wronskian. You should know the properties and definition of the Wronskian.

Section 2.4 shows all the previous theory works for complex equations. As such most of the proofs are simply restatements.

Chapter 3

Most of this chapter deals with equations of the form $x' = Ax$. Much of the early work is showing that $e^A$ is well-defined and has nice properties. Then we work on constructing the Jordan Canonical form. You should know what the Jordan Canonical Form is. How to find it and what that tells us about $e^A$.

The end of the chapter introduces the notion of integral for an equation. You should know the definition and how to verify a function is an integral.

Chapter 4

The midterm will only cover the first part of the chapter.

In Section 4.1 is introductory.

Section 4.2 introduces the notions of stability. You should know the definition of (positively) stable, asymptotically stable, and omega limit set. You should be able to prove basic facts about the omega limit set.

Section 4.3 states Lyapunov’s Theorems. You should know the statement of the first 3 theorems and how they can be used as in Section 4.4. You don’t need to memorize the proofs in Section 4.5, but it is good to know the ideas used.

The last section on the exam is Section 4.5. You should know the main statement Theorem 4.21 and how it can be used to determine the omega limit set of an orbit and asymptotic stability for a fixed point.