1. Let $F : \mathbb{R}^{n+1} \to \mathbb{R}$ be $C^r$ for some $r \geq 1$. Assume that $c \in \mathbb{R}$ such that $df_p \neq 0$ for all $p \in F^{-1}(c)$. Prove that $F^{-1}(c)$ is a smooth ($C^r$) manifold.

2. Let $M$ be a $k$-dimensional smooth manifold and $S(M)$ be point $(x, v) \in TM$ such that $|v| = 1$. Prove that $S(M)$ is a $2k - 1$-dimensional subbundle of $TM$ called the sphere bundle of $M$.

3. Let $G : \mathbb{R}^2 \to \mathbb{R}^4$ be given by

$$G(x, y) = ((r \cos y + a) \cos x, (r \cos y + a) \sin x, r \sin y \cos \frac{x}{2}, r \sin y \sin \frac{x}{2}).$$

Show this gives an embedding of the Klein bottle. (Hint: See what $G$ does to the square $[0, 2\pi] \times [0, 2\pi]$)

4. Show that $f : S^1 \to \mathbb{R}^2$ given by $f(t) = (\sin(2t) \cos t, \sin(2t) \sin t)$ is an immersion. Explain why $f(S^1)$ is not a submanifold of $\mathbb{R}^2$. 

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