1. Let $(V, \| \cdot \|)$ be a Banach space. Prove that

$$\| x \| - \| y \| \leq \| x - y \|$$

for all $x, y \in V$.

2. Find the logical blunder in the following “proof” of the completeness of $\mathbb{R}^n$. Let $\mu$ be the counting measure on $X = \{1, ..., n\}$. The Reisz-Fischer Theorem implies $L^2(X, \mu)$ is complete. But $L^2(X, \mu) = \mathbb{R}^n$.

3. Let $(V, \langle \cdot, \cdot \rangle)$ be a Hilbert space. A continuous linear function $T : V \to V$ is called an operator and is unitary if $T$ is a bijection and $\langle Tx, Ty \rangle = \langle x, y \rangle$ for all $x, y \in V$. Prove that if $T : V \to V$ is an operator that is onto and $\| Tx \| = \| x \|$ for all $x \in V$, then $T$ is unitary.