1. Prove that every nonempty open subset of $\mathbb{R}$ can be expressed as a countable disjoint union of open intervals $G = \bigcup_k (a_k, b_k)$ where $k$ can range over a finite or infinite set. Furthermore, show that this expression is unique except for the labelling. **Hint:** For any $x \in G$, show that there exists a largest open interval $A_x$ such that $x \in A_x$ and $A_x \subset G$.

2. In the notation of the above problem show that $\lambda(G) = \sum_k (b_k - a_k)$.

3. The above approach unfortunately does not generalize to higher dimensions. However, we can prove the following: Every nonempty open subset of $\mathbb{R}^n$ can be expressed as a countable union of non overlapping special rectangles that may be taken to be cubes. **Hint:** First cover $\mathbb{R}^n$ with cubes of side length 1. Select the cubes contained in $G$. Then bisect the sides of the remaining cubes to obtain cubes with side 1/2. Select the cubes of size 1/2 in $G$ and repeat.

4. Let $\epsilon > 0$. Prove that there is an open set $G$ containing $\mathbb{Q}$ such that $\lambda(G) < \epsilon$. 
