Review for Math 541 Final Exam

The final is comprehensive.

Definitions to know:

1. Special Rectangle (p. 24)
2. Special Polygon (p. 24)
3. Outer measure of a set (p. 35)
4. Inner measure of a set (p. 35)
5. Measure of a set with finite outer measure (p. 38)
6. Lebesgue measurable sets (p. 41)
7. Algebra (p. 54)
8. σ-algebra (p. 54)
9. Borel sets (p. 58)
10. Null set (p. 58)
11. Measurable function (p. 63)
12. Simple functions (p. 69)
13. The integral of a nonnegative measurable function (p. 76)
14. Integrable function (p. 82)
15. Almost everywhere (p. 86)
16. Measure space (p. 94)
17. Riemann integral (p. 108)
18. l-norm (p. 115)
19. p-norm (p. 134)
20. norm (p. 135, 141)
21. Holder conjugate (p. 136)
22. Banach space (p. 142)
23. Inner product (p. 149)
24. Hilbert space (p. 149)
25. infinity norm (p. 153)
26. Convolution (p. 158)
27. Fourier Transform (p. 166)
28. Periodic functions (p. 189)
29. Trigonometric series (p. 191)
30. Fourier coefficients (p. 194)
31. Fourier series (p. 197)

You should be able to:

1. Know properties proved in the construction of the Lebesgue measure (you may also be asked to reprove some of these facts)
2. Know properties of inner and outer measure and the approximation results for measurable sets.
3. Know basic facts about Cantor sets and how to use them.
4. Know the existence and example of nonmeasurable sets.
5. Determine if a collection of sets is an algebra or σ-algebra.
6. Know equivalent definitions of measurable functions (you may be asked to reprove one).
7. Work with general measure spaces, prove a function is measurable for such a space, and how to integrate.
8. Reprove basic facts, lemmas, corollaries, or results proven in the notes.
9. Show how to adapt arguments to a more general case (slight adaptation of results from the notes).
10. State Lebesgue’s increasing convergence theorem (p. 77) and consequences
11. State Fatou’s lemma (p. 80) and consequences
12. State Lebesgue’s dominated convergence theorem (p. 84) and consequences
13. Be able to use the fact that a function is Riemann integrable if and only if it is continuous almost everywhere
14. Be able to use the fact that any L1 function can be approximated both by continuous and smooth functions with compact support.
15. Be able to use the fact that translation is continuous for L1 functions.
16. Know the statement of Fubini’s Theorem for Nonnegative Functions and its consequences
17. Know the statement of Fubini’s Theorem for Integrable Functions and the statement of the theorem using Fubini’s Theorem for Nonnegative Functions.
18. Apply Fubini’s Theorem to compute integrals.
19. Know Holder’s inequality (p. 136).
20. Know Minkowski’s Inequality (p. 139).
22. Know Schwarz Inequality (p. 151).
23. Know basic facts about Lp spaces and their relations to one another
24. Know properties of convolutions and how to compute convolutions.
25. You should also be able to reprove basic facts about convolutions.
26. Know definition and basic properties of the Fourier Transform as well as how to compute the transform.
27. Know the Fourier Inversion Theorem (p. 176)
28. Know the properties of the Schwarz class
29. Know Parseval’s Identity (p. 184)
30. Know the definition of the Fourier-Plancherel Transform and properties
31. Compute Fourier coefficients for examples
32. Know basic properties of convergence for Fourier series