Review for Math 541 Midterm 1

The midterm covers from the beginning to page 103 in the notes (up until the Riemann integral).

**Definitions to know:**
1. Lebesgue number (p. 12)          11. $\sigma$ – algebra (p. 54)
2. Relatively open (p. 16)           12. Borel sets (p. 58)
3. Urysohn function (p. 22)         13. Null set (p. 58)
4. Special Rectangle (p. 24)         14. Measurable function (p. 63)
5. Special Polygon (p. 24)           15. Simple functions (p. 69)
6. Outer measure of a set (p. 35)    16. The integral of a nonnegative measurable function (p. 76)
7. Inner measure of a set (p. 35)    17. Integrable function (p. 82)
8. Measure of a set with finite outer measure (p. 38) 18. Almost everywhere (p. 86)
9. Lebesgue measurable sets (p. 41)  19. Measure space (p. 94)
10. Algebra (p. 54)                  20. Radon-Nikodym derivative (p. 103)

**You should be able to:**

State and prove the following theorems:
1. Lebesgue’s increasing convergence theorem (p. 77)
2. Fatou’s lemma (p. 80)
3. Lebesgue’s dominated convergence theorem (p. 84)

**You should be able to:**

1. Know and prove basic facts from Analysis such as open sets, compact, sets, open covers, continuity, etc
2. Know properties proved in the construction of the Lebesgue measure (you may also be asked to reprove some of these facts)
3. Know properties of inner and outer measure and the approximation results for measurable sets.
4. Know basic facts about Cantor sets and how to use them.
5. Know the existence and example of nonmeasurable sets.
6. Determine if a collection of sets is an algebra or $\sigma$-algebra.
7. Know equivalent definitions of measurable functions (you may be asked to reprove one).
8. Work with general measure spaces, prove a function is measurable for such a space, and how to integrate.
9. Reprove basic facts, lemmas, corollaries, or results proven in the notes.
10. Show how to adapt arguments to a more general case (slight adaptation of results from the notes).