Find the line of intersection of the two planes

\[ P_1: x + y + z = -1, \quad P_2: 2x - 3y + 2z = 2 \]

Answer:
For the line we need

a) a direction vector \( \vec{u} = \langle a, b, c \rangle \).

b) a point on the line, \( P_0(x_0, y_0, z_0) \)

a) To obtain a direction vector \( \vec{u} \)

\[ \vec{u} = \vec{n}_1 \times \vec{n}_2 = \langle 1, 1, 1 \rangle \times \langle 2, -3, 2 \rangle \]

or

\[ \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & -3 & 2 \end{vmatrix} = 5\hat{j} - 5\hat{k} = \langle 5, 0, -5 \rangle \]

b) To obtain a point on \( L \)

\[ x = 0 \Rightarrow (y + z = -1) + 3 \]

\[ -3y + 2z = 2 \]

\[ 0 = 5z \Rightarrow z = -1/5 \Rightarrow y = -1 - 2 = -4/5 \]

or \( \langle x_0, y_0, z_0 \rangle = \langle 0, -4/5, -1/5 \rangle \)

Then \( L: \langle x, y, z \rangle = t \langle 5, 0, -5 \rangle + \langle 0, -4/5, -1/5 \rangle \)
#73) Find the distance between the parallel planes

\[ \Pi_1: 2x - 3y + z = 4 \quad \text{and} \quad \Pi_2: 4x - 6y + 2z = 3 \]

\[ \text{Answ: To compute the distance, we need to:} \]

\[ \text{a) obtain a perpendicular line to both planes} \]

\[ \text{b) determine the two points in which this line L intersects the two planes.} \]

\[ \text{c) compute the distance between } P_0 \text{ and } P_1. \]

First, a point in \( \Pi_1 \) is \( \langle 0, 0, 4 \rangle \)

Second, a line perpendicular to \( \Pi_1 \) and \( \Pi_2 \) through \( \langle 0, 0, 4 \rangle \) is

\[ L: \langle x, y, z \rangle = t \langle 2, -3, 1 \rangle + \langle 0, 0, 4 \rangle. \]

Third, intersection with plane \( \Pi_2 \):
\[ L: x = 2t, \quad y = -3t, \quad z = t + 4, \quad \text{parametric form} \]

Substitute into equ. for \( \Pi_2 \)

\[ 4(2t) - 6(-3t) + 2(t+4) = 3 \]

\[ 8t + 18t + 2t + 8 = 3 \Rightarrow 28t = -5 \Rightarrow t = -\frac{5}{28} \]

\[ \Rightarrow x = -\frac{5}{14}, \quad y = \frac{15}{28}, \quad z = -\frac{5}{28} + 4 = \frac{112 - 5}{28} = \frac{107}{28} \]

Therefore, point of intersection of line \( L \) with plane \( \Pi_2 \)

\[ \left( -\frac{5}{14}, \frac{15}{28}, \frac{107}{28} \right) \]

Fourth, distance from \( (0,0,4) \in \Pi_1 \), to \( \left( -\frac{5}{14}, \frac{15}{28}, \frac{107}{28} \right) \) in \( \Pi_2 \)

\[ d = \sqrt{\frac{25}{(14)^2} + \left(\frac{15}{28}\right)^2 + \left(\frac{107}{28} - 4\right)^2} \]

\[ = \sqrt{\frac{25}{196} + \frac{225}{784} + \frac{107^2}{28^2}} = \sqrt{\frac{25}{196} + \frac{225}{784} + \frac{25}{784}} \]

\[ = \sqrt{\frac{25}{196} + \frac{250}{784}} = \sqrt{\frac{125}{392}} = \frac{5\sqrt{5}}{2\sqrt{198}} = 0.6682. \]