Exercise 1.5 (d) (i)

The grid consists of the points

\[ x(1) = 0.4 \]
\[ x(2) = 0.5 \]
\[ x(3) = 0.6 \]

\[ xb = 0.5 \]

Finite difference stencil of the second derivative of an arbitrary smooth function \( u \) at \( x = xb \) based on the above grid points

\[ (100) * u(x(1)) + (-200) * u(x(2)) + (100) * u(x(3)) \]

Leading order error terms for smooth \( u \) at \( xb = 0.5 \) using \( n = 3 \) points

\[ LTE = 0 + u^{(n)}(xb) + (1/1200) * u^{(n+1)}(xb) + ... \]

Application of the above finite difference to approximate the derivative \( u^{(2)} \) of the input function \( u = e^{x/3} \) at \( xb = 0.5 \) based on the grid "xpts":

Approximation of \( u^{(2)}(0.5) = 0.131274 \)

Exact value of \( u^{(2)}(0.5) = 0.131262 \)

Actual Error = 1.21544e-05

Estimated Error = 1.21539e-05

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