Math 315-003

Test 3

Name____________________

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Show relevant work!

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1. State and prove the First Fundamental Theorem of Calculus.

2. Suppose the function $f : \mathbb{R} \to \mathbb{R}$ is continuous. Define

$$G(x) = \int_0^x (x - t)f(t)dt$$

for all $x$. Prove that $G''(x) = f(x)$ for all $x$.

3. Describe in words the $n^{th}$ degree Taylor Polynomial for a function $f$ at the point $x_0$. Explain why it is uniquely determined.

4. Show the number $e$ is irrational.
5. Show that the Taylor expansion of \( f(x) = \sin(x) \) at \( x_0 = 0 \) converges for all points \( x \).

6. Suppose that \( \sum_{k=1}^{\infty} a_k \) converges. Show \( \lim_{n \to \infty} a_n = 0 \).

7. Define what it means for a sequence to be Cauchy and show that a convergent sequence is Cauchy.

8. Suppose \( \sum_{k=1}^{\infty} a_k \) and \( \sum_{k=1}^{\infty} b_k \) are series of positive numbers such that \( \lim_{k \to \infty} \left( \frac{a_k}{b_k} \right) = \lambda \) and \( \lambda > 0 \). Show that \( \sum_{k=1}^{\infty} a_k \) converges if and only if the series \( \sum_{k=1}^{\infty} b_k \) converges.
9. For a number $r$ such that $|r| < 1$, show $\sum_{k=1}^{\infty} r^k$ converges.

10. Does the series $\sum_{k=1}^{\infty} \frac{1}{(k+1)\ln(k + 1)}$ converge? Prove your assertion.