1. Show that if \( \lim a_n = a \) and \( \lim b_n = b \), then \( \lim a_n b_n = ab \).

2. Show that the Nested Interval Property implies the Axiom of Completeness.

3. Show that if \( 0 < r < 1 \), then \( \lim r^n = 0 \).

4. Define what it means for a sequence to be Cauchy and show that a Cauchy sequence converges.
5. Prove that a closed interval has the property that if it covered by a collection of open sets, then some finite sub-collection of the open sets covers.

6. If $f$ and $g$ are differentiable functions with domain all real numbers. If $g(x) \neq 0$, prove that $F(x) = \frac{f(x)}{g(x)}$ is differentiable at $c$ and find the derivative.

7. Prove the uniform limit of continuous functions is continuous.
8. Show that if \( f \) is a differentiable, real-valued function defined for all real numbers with a bounded derivative, then \( f \) is uniformly continuous.

9. Prove that if a power series \( \sum_{n=0}^{\infty} a_n x^n \) converges at some point \( x_0 \), then it converges absolutely for any \( x \) satisfying \( |x| < |x_0| \).

10. Prove that if a power series \( \sum_{n=0}^{\infty} a_n x^n \) converges for all \( x \in (-R,R) \), then the differentiated series \( \sum_{n=1}^{\infty} na_n x^{n-1} \) converges as well for all \( x \in (-R,R) \).

11. If \( P_n(x) \) is the \( n \)th degree Taylor polynomial for \( f(x) \), and \( f(x) = P_n(x) + E_n(x) \), find \( E_n(0), E_n^{(1)}(0), E_n^{(2)}(0), E_n^{(3)}(0), \ldots, E_n^{(n)}(0) \) and a formula for \( E_n^{(n+1)}(x) \). Explain your reasoning. ( \( E_n^{(k)}(x) \) is the \( k \)th derivative of \( E_n(x) \).)
12. Define the lower and upper integrals of a function.

13. Prove that if a function $f$ is continuous on a closed interval $[a, b]$, then it is integrable.

14. State and prove one part (you choose) of the Fundamental Theorem of Calculus.
15. Assume that the functions $f$ and $|f|$ are integrable on the interval $[a, b]$. Show \[ \left| \int_a^b f \right| \leq \int_a^b |f| \, dt. \]

16. Let $f$ be a differentiable function defined on $[a, b]$ so that the derivative $f'$ of $f$ is continuous. If $P = \{x_0, x_1, x_2, \ldots, x_n\}$ is a partition of $[a, b]$, show \[ \sum_{k=1}^{n} \left| f(x_k) - f(x_{k-1}) \right| \leq \int_a^b |f| \, dt. \]