1. Find the surface area when the line segment from (4, 0) to (16, 5) is rotated about the y-axis.

Use the formula for the surface area of a lampshade \( SA = 2\pi \frac{R_1 + R_2}{2} \ell \) where \( R_1 \) and \( R_2 \) are the two radii and \( \ell \) is the slant height. In this case \( R_1 = 4, R_2 = 16 \) and \( \ell = 13 \).

The answer is \( 260\pi \).

2. The curve \( y = \sqrt{4 - x^2}, -1 \leq x \leq 1 \), is rotated about the x-axis. Find the area of the resulting surface.

\[
\frac{dy}{dx} = \frac{-x}{\sqrt{4 - x^2}}.
\]

The surface area is \( 2\pi \int_{-1}^{1} \sqrt{4 - x^2} \sqrt{1 + \frac{x^2}{4 - x^2}} \, dx \)

\[
= 2\pi \int_{-1}^{1} \sqrt{4 - x^2} \sqrt{\frac{4}{4 - x^2}} \, dx = 4\pi \int_{-1}^{1} 1 \, dx = 8\pi
\]

3. Find the centroid of the following system consisting of a square and an isosceles triangle.

\[
A = 9 + 3 = 12
\]
\[
M_y = -\frac{3}{2} \cdot 9 + 1 \cdot 3 = -\frac{21}{2}
\]
\[
M_x = \frac{3}{2} \cdot 9 + 1 \cdot 3 = \frac{33}{2}
\]
\[
\bar{x} = \frac{M_y}{A} = -\frac{7}{8}
\]
\[
\bar{y} = \frac{M_x}{A} = \frac{11}{8}
\]

4. Find the centroid of the region between the two triangles in the x-y plane. You may use either Hint 1 or Hint 2. Hint 1: The area can be found as the difference of two areas. In a similar manner, the moment about the x-axis can be found as the difference of two moments. Hint 2: Use the Theorem of Pappus.

\[
A = 4 - 1 = 3
\]
\[
M_x = \frac{2}{3} \cdot 4 - \frac{1}{3} \cdot 1 = \frac{7}{3}
\]
\[
\bar{y} = \frac{M_x}{A} = \frac{7}{9}
\]
\[
\bar{x} = 0 \text{ by symmetry}
\]
5. Evaluate the following limits if they exist. If the limit does not exist, so state.

(a) \( \lim_{n \to \infty} \frac{1}{n} = 0 \)

(b) \( \lim_{n \to \infty} \left(1 + \frac{5}{n}\right)^n = e^5 \)

(c) \( \lim_{n \to \infty} \frac{\sqrt[n]{n^5 + 2n^3 + 5}}{n^3} = 0 \)

6. Define \( \sum_{n=1}^{\infty} a_n = L \). Let \( S_n = a_1 + a_2 + a_3 + \cdots + a_n \). Then \( \sum_{n=1}^{\infty} a_n = L \) means that \( \lim_{n \to \infty} S_n = L \).

7. What is the hydrostatic force on the given plate whose top is at the surface of the water if the density of water is \( \delta \) lbs/ft\(^3\)?

![Diagram of plate with dimensions and depth](image)

Depth of centroid of \( 2 \times 4 \) rectangle = 1 ft.
Depth of centroid of \( 2 \times 3 \) rectangle = \( \frac{7}{2} \) ft.
Depth of centroid of triangles = 3 ft.
Force = \([1 \cdot 8 + \frac{5}{2} \cdot 6 + 3 \cdot (\frac{3}{2} + \frac{3}{2})]\) \( \delta \) lbs
= 38\( \delta \) lbs

8. What is the hydrostatic force on a 2 foot by 2 foot square diamond aquarium window whose top is 2 feet below the surface of the water if the density of water is \( \delta \) lbs/ft\(^3\)?

![Diagram of diamond window with dimensions and depth](image)

Diagonal of square = \( 2\sqrt{2} \) ft. Centroid of square is \( 2 + \sqrt{2} \) ft below the surface. Area of square = 4 ft\(^2\). Force = \((2 + \sqrt{2}) \cdot 4\delta \) lbs = \((8 + 4\sqrt{2})\) \( \delta \) lbs
9. If $0 < r < 1$, prove that $\lim_{n \to \infty} r^n = 0$. Multiply each side of the inequality by $r^n$ to get $0 < r^{n+1} < r^n$. So the sequence $r^n$ is decreasing and bounded below by 0. So by a theorem, it must converge to some number $L$. But $\lim_{n \to \infty} r^{n+1} = L$ and $\lim_{n \to \infty} r^n = rL$. So $rL = L$. Since $r \neq 1$, we must have $L = 0$.

10. Find the fifteenth partial sum $S_{15}$ for the series $\sum_{n=1}^{\infty} (-1)^{n+1}$.

An even partial sum has the same number of 1’s and −1’s so the partial sum is 0. An odd partial sum has one more = 1 than −1 so the partial sum is 1.

11. Determine whether each series converges or diverges. If it converges, give its sum.

(a) $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}} = \phantom{2}$ Diverges because the terms go to $1 \neq 0$.

(b) $\sum_{n=1}^{\infty} \frac{2}{4n^2 - 1} = \phantom{2}$ Use partial sum decomposition to get $\frac{2}{4n^2 - 1} = \phantom{2}$

Which is seen to be $\frac{1}{2n - 1} - \frac{1}{2n + 1}$. The series is now seen to be telescoping $1 - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \cdots$ its sum is 1.

(c) $\sum_{n=1}^{\infty} \frac{2^{n+1}}{3^n} = \phantom{2}$

This is a geometric series with first term $\frac{4}{3}$ and ratio $\frac{2}{3}$. The sum is $\frac{4}{1 - \frac{2}{3}} = 4$.
12. Determine whether each series converges or diverges. State any convergence/divergence tests you use. For the Integral Test, evaluate the appropriate integral. For the Comparison Test or Limit Comparison Test give the appropriate comparison series.

(a) \[ \sum_{n=1}^{\infty} ne^{-n^2} \] Converges by Integral Test or Ratio Test. Check details.

(b) \[ \sum_{n=1}^{\infty} \frac{\ln n}{n^3} \] Converges by Integral Test or by Comparison Test with the series \[ \sum_{n=1}^{\infty} \frac{1}{n^2} \] since \( \ln n < n \) for large \( n \). Check details.

(c) \[ \sum_{n=1}^{\infty} \frac{n^2 + 3n + 1}{n^3 + 2n^2 + n + 1} \] Diverges by Limit Comparison Test with the harmonic series. Check details.

(d) \[ \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3 + 2n^2 + n + 1}} \] Converges by either the Limit Comparison Test or the Comparison Test with the series \( \sum_{n=1}^{\infty} \frac{1}{n^{3/2}} \). Check details.

(e) \[ \sum_{n=1}^{\infty} \frac{\sin\left(\frac{1}{n}\right)}{\sqrt{n}} \] Converges by Limit Comparison test with the series \( \sum_{n=1}^{\infty} \frac{1}{n^{3/2}} \). Check details.